Topology Optimization of Flow Fields With Multi-physics Phenomena Using Lattice Boltzmann Method (LBM)

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Introduction

This work presents numerical results of topology optimization problem using lattice Boltzmann method (LBM) 2D9V model and level set method (LSM) to express the distribution of material itself. Based on previous researches, the governing algorithms are constructed which is concerned with finding optimal geometry of fluidic devices. Specifically, with the employment of the no-slip condition, prescribed velocity and prescribed pressure boundary to solve a problem of maximizing the kinetic dissipation energy, the proposed method is validated to be able to work with different constraints and conditions. In this study, the validation of variable velocity inlet varying the related flow rate crossing the design domain is granted. To be more specific, the inlet constraint employs sinusoidal velocity equation as inlet condition. In other words, the velocity inlet plays an important role in the optimal topology resulting to particular cavities. In this research, we first build up a methodology with Boltzmann equation (BE) and lattice Boltzmann equation (LBE) are presented as the fluid solver, then an optimization based on level set method and optimization algorithm are carried out correspondingly. Likewise, the numerical implementation is performed to valid the proposed method for fluid problems with different constraints and different conditions. More specifically, the numerical example is done with the topology optimization concerned the effect of velocity inlet boundary limited in unsteady incompressible flow conditions. Finally, the judgment of the effect of velocity inlet condition on topology optimization proceeding is delineated visually though the numerical results. This study is not only requires a solution to flow problem for given velocity boundary condition but also a prediction on how the initial design affects the flow.

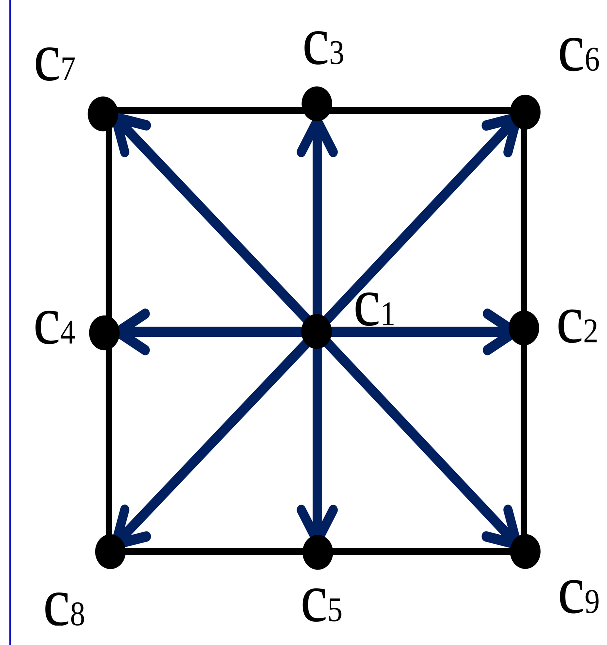


Figure 1. 2D9V lattice Boltzmann model

Table 1. Lattice velocities and weight parameters in each direction

Lattice direction	Lattice velocity	Weight parameter
1	(0,0)	4/9
2	(1,0)	1/9
3	(0,1)	1/9
4	(-1,0)	1/9
5	(0,-1)	1/9
6	(1,1)	1/36
7	(-1,1)	1/36
8	(-1,-1)	1/36
9	(1,-1)	1/36

Methodology

Objectives: The purpose of this research is to maximize the dissipation kinetic energy of the internal flow.

$$J=\int \rho\left[u_k\left(\frac{1}{2}u_i^2+p\right)-\frac{1}{\mathrm{Re}}u_i\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)\right].n_k dS$$

$$+\frac{1}{\mathrm{Re}}\int \rho\frac{\partial u_i}{\partial x_k}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)d\Omega$$

The objective function is evaluated after certain of time in steady state condition, the objective function and constraints are given by:

$$\inf_{\phi} J\big|_{t=t_1}=\int_D A(\rho,u)d\Omega+\int_\Gamma B(\rho,u)d\Gamma,$$

s.t.

$$G=\int_I\int_D\int_\Xi g\left\{\frac{\partial f_i}{\partial t}+\xi\cdot\frac{\partial f_i}{\partial x}=-\frac{1}{\lambda}\cdot\left(f_i-f_i^{eq}\right)\right\}d\xi d\Omega dt,$$

$$V=\int_D \chi_\phi d\Omega-V_{\max}\le 0,$$

where, J represents the objective function at the steady state, the equation of is the weak form of Boltzmann equation, V is volume constraint, which is permissible to maximal volume of the fluid domain.

Governing equations: The velocity discretization in time and space set along ith lattice direction of the Boltzmann equation, with the Bhatnagar-Gross-Krook (BGK) approximation is acknowledged, the governing equation of flow field is

$$\frac{\partial f_i}{\partial t}+\xi\cdot\frac{\partial f_i}{\partial x}=-\frac{1}{\omega}\cdot\left(f_i-f_i^{eq}\right).$$

The above equation includes the two processes, propagation or streaming process placed in the left hand side, and the other is collision process placed in the other side. The equilibrium is given by:

$$f^{eq}=\frac{\rho}{\left(2\pi RT\right)^{d/2}}\exp\left(-\frac{|\xi-u|^2}{2RT}\right),$$

By tackling the Maxwell-Boltzmann equation by the Taylor series expansion with respect to space and time, the discrete lattice Boltzmann equation is expressed as follows :

$$f_i\left(x+c_ih,t+\Delta t\right)-f_i\left(x,t\right)=\frac{1}{\omega}\left(f_i\left(x,t\right)-f_i^{eq}\left(x,t\right)\right).$$

The local equilibrium distribution function is given by the following formula utilizing the Taylor expansion up to second order accuracy

$$f_i^{eq}=\rho\cdot w_i\cdot\left[1+3h^2\cdot\xi_i\cdot u+\frac{9}{2}h^4\cdot(\xi_i\cdot u)^2-\frac{3}{2}h^2\cdot u\cdot u\right].$$

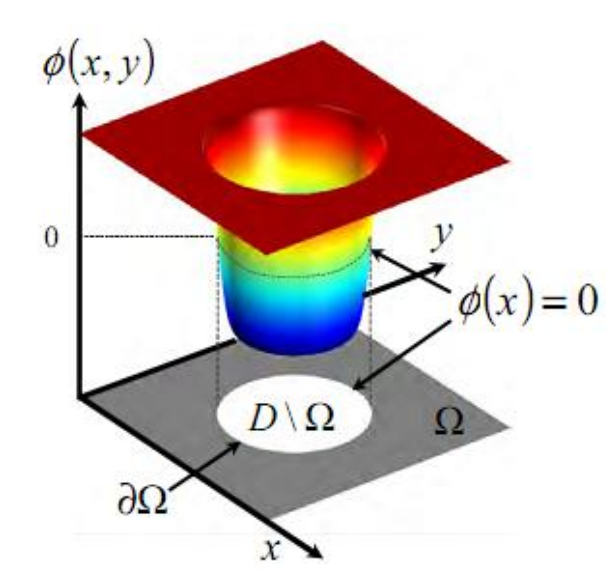
Furthermore, the macroscopic fluid velocity and density are obtained from mass and momentum laws in lattice scheme and pressure represents as the follows:

$$\rho=\sum_{i=1}^{q}f_i,\quad u=\frac{1}{\rho}\sum_{i=1}^{q}\xi_i f_i,\quad p=\frac{\rho}{3}\ .$$

Finally, the prescribed velocity and pressure boundary conditions are applied to all nodes on the left-inlet and right-outlet boundaries, while the no-slip boundary condition is considered on solid boundaries invariably.

Level Set Method: The numerical technique for tracking interfaces and shapes. It can be formulated using a fixed design domain D, which consists of a solid domain Ω, structural boundaries ∂Ω and a void domain D\Ω. The Level Set Method makes it very easy to follow shapes that change topology, for example:

- Shape splits in two.
- Develops holes.
- The reverse of previous operations.



$$\begin{cases}1\ge\phi(x)>0 & \text{for}\quad \forall x\in\Omega\setminus\partial\Omega\\ \phi(x)=0 & \text{for}\quad \forall x\in\partial\Omega\\ 0>\phi(x)\ge -1 & \text{for}\quad \forall x\in D\setminus\partial\Omega\end{cases}$$

Results

Table 2. Initial conditions of numerical example

Parameters			
Lattice points	32x32	Reynolds number	$\mathrm{Re}=3$
$V_{\max}/V_0$	0.6	Relaxation time	$\omega=0.8$
ρ_{out}	3.0	Kinematic viscosity	$\nu=1\times10^{-1}$
Vel_{in}	$Vel_{in}=U_0Sin(\omega t)$	Velocity U_0	$U_0=1\times10^{-2}$

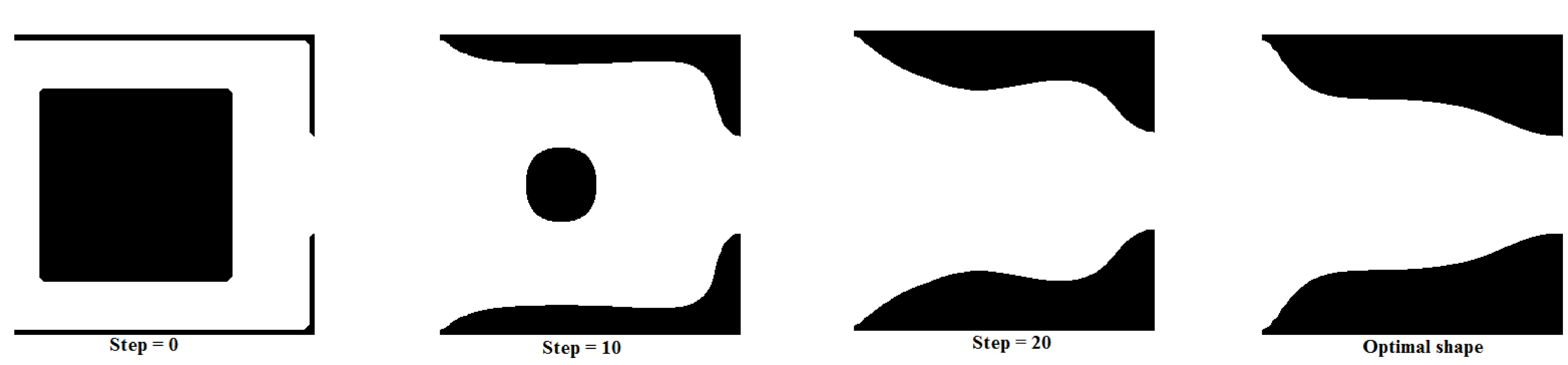


Figure 2. Optimization proceeding.

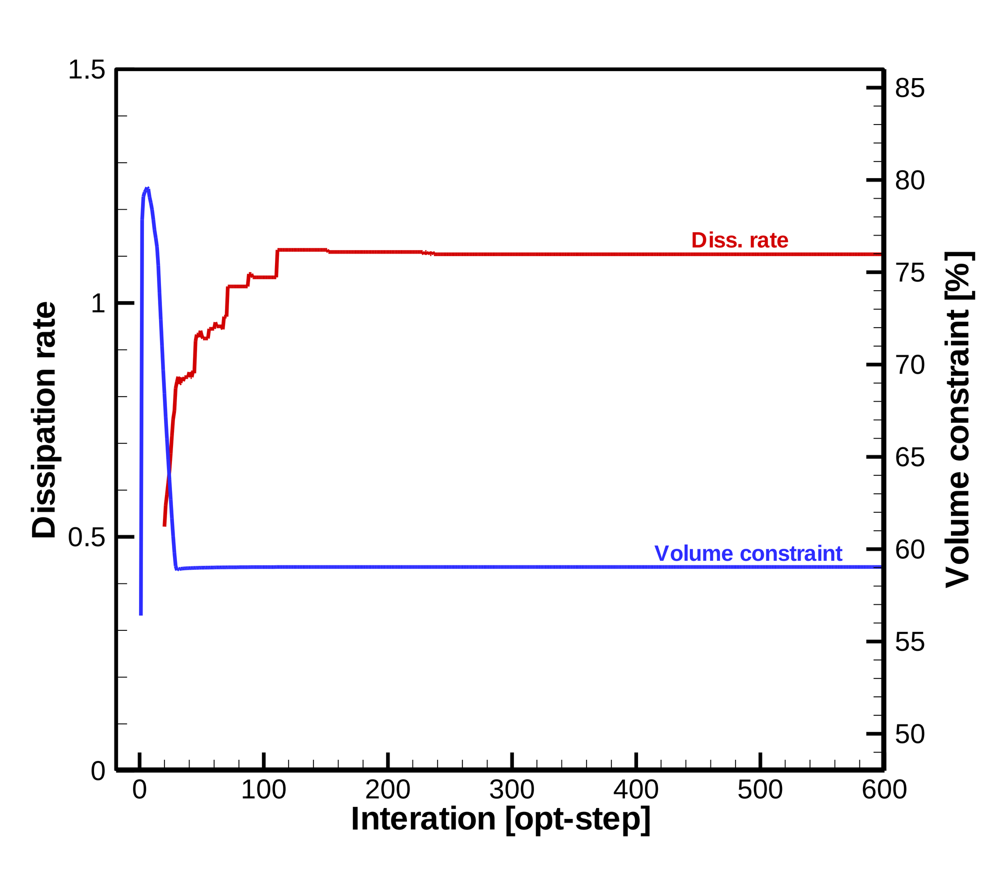


Figure 3. Objective value and volume const. evolution.

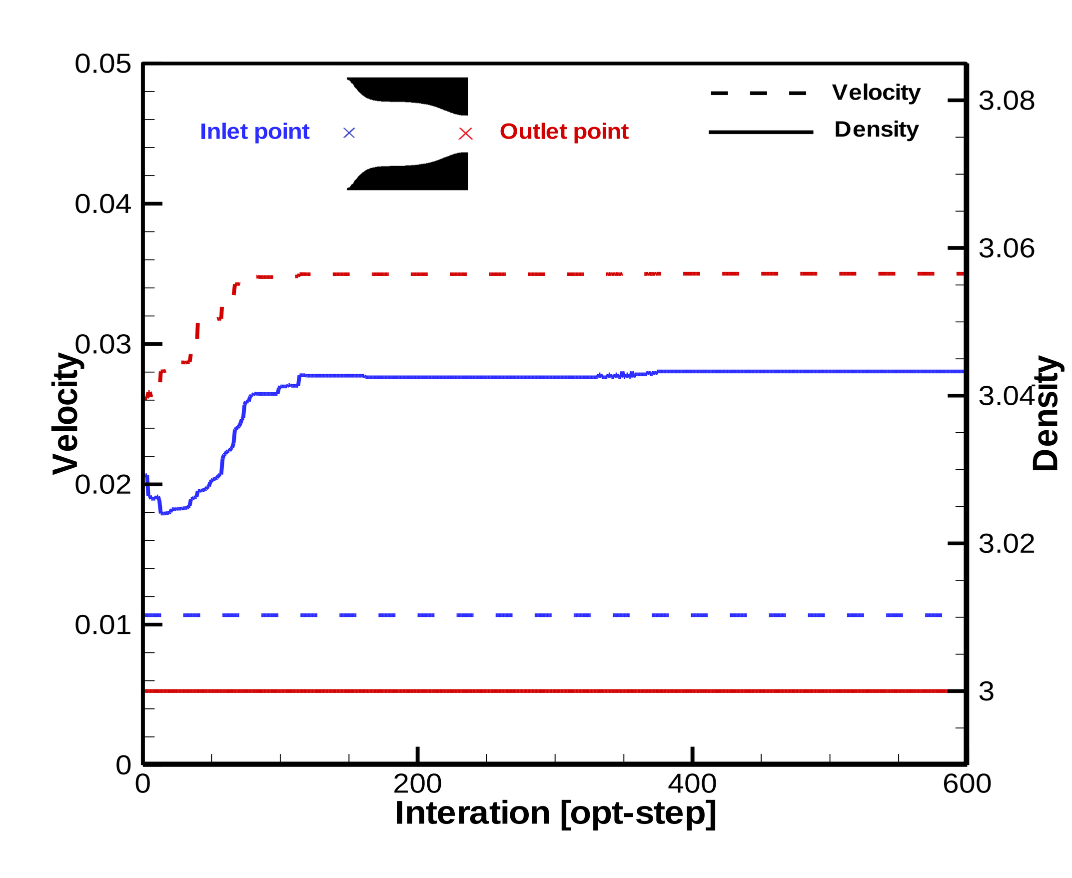


Figure 4. Velocity and density evolution.