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A Parallel Implementation of the Fast Multipole Boundary Integral Equation Method in Elastostatic Crack Problems in 3D

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This paper discusses a parallel implementation of the new Fast Multipole Method (FMM) in three-dimensional elastostatic crack problems. The proposed approach distributes exponential components in the six lists of the new FMM to different processors. An MPI based implementation is made. The efficiency of the new implementation is demonstrated via numerical examples.

Key Words: Parallelisation, New FMM, Elastostatics

1. Introduction

The Fast Multipole Method (FMM) is now well recognised as an efficient method for enhancing the performance of Boundary Integral Equation Methods (BIEM) in various applications[(1)]. Particularly effective is the application of FMM in problems of the Laplace type, including elastostatics, where the original FMM proposed by Rokhlin and Greengard [(2, 3)] is considered sufficiently effective for many practical problems. In order to further improve the efficiency of FMM, Hrycak and Rokhlin [(4)] proposed a new FMM for the two-dimensional Laplace equation, Greengard and Rokhlin [(5)] and Cheng et al. [(6)] for the three-dimensional Laplace equation, and Greengard et al. [(7)] for the three-dimensional Helmholtz equation. These approaches are characterised by the 'diagonal' M2L operations which have no instability problems in contrast to the conventional diagonal forms for Laplace's equation [(8)]. The new FMM is considered to be more efficient than the original FMM because the complexity of the M2L of the new FMM is $O(p^3)$ while that of the original FMM is $O(p^4)$, where p is the number of terms kept in the truncated multipole expansion. The superiority of the new FMM has been demonstrated by Yoshida et al. [(9)] in crack problems for the Laplace equation in 3D. They also extended the new FMM to elastostatics in Yoshida et al.[(10, 11)].

In this paper we attempt to further improve the efficiency of the new FMM in elastostatic crack problems in 3D by parallelising the algorithm. Our interest in this paper is to speed up the calculation, rather than to increase the size of the problem one can handle. The parallelisation is achieved by assigning only one of the 6 types of the exponential expansions to one processor. We shall see that the proposed implementation improves the execution time by 4 to 5 times compared to the sequential version of the new FMM.

In this paper we shall use both indicial and direct notations for tensorial quantities. The summation convention is used for repeated indices. Also the position vector of a point x will be denoted by either $\vec{x}$ or $\overrightarrow{Ox}$, the latter being the preferred notation when one needs to show the origin explicitly.

2. Crack Problems and Integral Equations

Let $S \subset \mathbb{R}^3$, or a 'crack', be a union of smooth non-self-intersecting curved surfaces having smooth edges ∂S. Also let $\vec{n}$ be the unit normal vector to S. Our problem is to find a solution $\vec{u}$ of the equation of elastostatics

$$C_{ijkl}u_{k,lj} \quad = \quad 0 \quad \text{in } \mathbb{R}^3 \setminus \overline{S}$$

subject to the boundary condition

$$t_i^{\pm} := C_{ijkl}u_{k,l}^{\pm}n_j = 0 \quad \text{on } S \tag{1}$$

regularity

$$\vec{\phi}(x) \quad := \quad \vec{u}^+(x) - \vec{u}^-(x) = 0 \quad \text{on } \partial S$$